

# The Feasibility of using Small-World Networks as models in Informatics, and in particular, the brain.

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## Abstract

This review investigates the concept of a small-world network, its feasibility and applications to the area of informatics, in particular the modelling of the brain. Approximately, a small-world network is a graph in which most nodes are not neighbours of one another, but where most nodes are a small number of hops or steps away from all others.

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# 1 Introduction

## 1.1 Motivation

A small-world network can be seen as one which combines the properties of a regular and random network. To analyse these small-world network, they are usually represented by concepts derived from graph theory (See appendix for formal definitions). A node is used to represent a certain property and edges connect these concepts appropriately. The structural properties of these graphs are characterised by their path length,  $L(p)$ , and clustering coefficient,  $C(p)$ . As suggested by their names, the path length measures the typical separation between two edges in the graph and the clustering coefficient the ‘cliquishness’ of a typical neighbourhood. Thus, a small-world network is a graph which has short path length but a high clustering coefficient.

It is due to the strong mathematical foundations of graph theory and the intuitive analogy of the way one thinks in categories linking to other categories like a small-world network that motivated the investigation into these small-world network. We look at how accurately these small-world network model certain networks in areas in Informatics such as the World-Wide Web (WWW) and in particular, the brain.

## 1.2 History

The work of Paul Erdős & Alfréd Rényi on random graphs in the 1960’s provided the basis model of a network which tends to be used for comparison methods. In many studies it is shown that the small-world model characterises phenomenon better than random network-based ones. Small-world networks were popularised by Stanley Milgram’s ‘small-world experiment’ in 1967 which implied the notion of the ‘six degrees of separation’ phenomenon. However, it was not until 1998 with Watts’ and Strogatz’ esteemed paper entitled ‘Collective dynamics of ‘small-world’ networks’ which garnered much interest and research into the area. In this paper, they introduced small-world network as a realisable alternative to previous categorisation of a network either being regular or random, with many studies which seemed to confirm this ‘goodness of fit’. Malaquais et al.[12] provided a good theoretical argument towards why a small-world architecture would work as a model of the human mind, in particular appealing to the way people categorise and

group concepts together, e.g. types of food. More concrete support for this is found in research by Postma et al.[15] which will be discussed later.

Another highly influential paper was published in 1999 by Barabási et al.[4] which criticises both the random and Watts' and Strogatz' model as incomplete - they lacked the incorporation of two major aspects of 'real' world networks. First, both models assume a network starts and ends with a fixed number of nodes. In reality, many real world networks are open and form by the continuous addition of new nodes throughout the lifetime of the network, e.g. the WWW grows with the addition of new webpages. Second, both models are wired and rewired randomly whereas many real networks exhibit preferential connectivity, e.g. a new webpage is more likely to include links to well-known pages with high connectivity. More importantly, Barabási et al. discovered that the inclusion of these two factors led to a power law distribution,  $P(k) \approx k^{-\gamma}$ . This finding is of particular interest because recent studies in networks in Informatics have shown to exhibit a power law distribution, e.g. for a functional network of activated voxels[7].

However, caution and interest also emerged through the mathematical analysis of this power law distribution as dealt with by Amaral et al.[3], more specifically the robustness of a power law distributed network against random error. One of the main features of these networks is the high occurrence of 'hubs', i.e. nodes with degrees that are significantly higher than the average. Since failures occur at random and the vast majority of nodes are those with small degree, the likelihood that a hub would be affected is almost negligible.<sup>1</sup> The caution is naturally in the event of removing one or more of these 'hubs'; the result would be a network consisting of a sparse graph which would give a less mathematically rich structure to the network and hence, less information. In short, scale-free networks are highly robust against errors, but potentially vulnerable to planned attack[3].

Thus, the history of small-world network arose from a sound and continually developing mathematical theory with practical applications to networks and can be seen to relate to some connectionist models of the brain. This already demonstrates a rich prospect of research in the theory of small-world network alone, without even knowing its applications. However, this is just one of the aspects of small-world network which generates interest. Our aim is to see how much potential there is for the brain to be modelled as a

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<sup>1</sup>[http://en.wikipedia.org/wiki/Scale-free\\_network](http://en.wikipedia.org/wiki/Scale-free_network)

small-world network and vice-versa.

## 2 Arguments For and Against

### 2.1 Examples of Effectiveness

We begin by looking at the initial effectiveness of small-world networks for modelling specific types of systems related to the brain. In Watts' and Strogatz'[21] seminal paper they give the influential example of the completely mapped neural network of the (nematode) worm, *C. Elegans*[1] where an edge joins two neurons if they are connected by either a synapse or a gap junction. This network exhibited small-world properties with  $L > L_{rand}$  and  $C \gg C_{rand}$ . A property of path length of a random network,  $L_{rand}$ , is that it is short, which small-world networks are also characterised by. Here, although it is seen that  $L > L_{rand}$ , the actual figures are  $2.65 > 2.25$  which is only 15% difference and for practical reasons, it is fairly justified to take  $L \approx L_{rand}$  (as we shall see later). Thus, we take the notion of  $C \gg C_{rand}$  (which is certainly justified since  $0.28 \gg 0.05$  here) along with  $L \approx L_{rand}$  as the properties of small-world networks.

Furthermore, one of the proposed future work areas from the paper was the plausibility of the brain having a small-world network structure. A study that showed small-world networks of coupled phase oscillators synchronising as readily as a standard model presented the possible relevance of it to the observed synchronisation of widely separated neurons in the visual cortex[9].

In 2006, Bassett and Bullmore[5] published a very promising paper entitled "Small-Brain Networks" which provided further support for using small-world models for understanding the structure and function of the brain. They presented results from studies of the visual cortex of the macaque monkey and the cat from several researchers and deduced small-world network structures from them. To quote: "*The first graphical analyses of mammalian cortical network, in the early 1990s, preceded the mathematical development of the small-world model but identified many features of anatomical connectivity that would later be recognized as compatible with it*".

In 1991, Felleman and Van Essen[8] compiled an anatomical connectivity matrix that summarised 305 axonal connections between 32 areas of the visual cortex in the macaque monkey. They found that on producing several levels of cortical processing by differentiating connections with regard to the

(laminar) structure of the cortex, results showed the existence of multiple separated parallel processing streams with fewer connections between the streams.

In particular, the segregation of the ventral and dorsal streams was confirmed by Young et al.[23] who had began considering anatomical connectivity matrices from tract-tracing studies of the macaque monkey and cat also. Their connectivity matrix was drawn from 301 connections and 30 areas in the visual cortex of the macaque monkey with connections being coded as 0 (weak/absent) to 2 (strong & reciprocal). Similarly, Scannell et al.[17][18] analysed results from tract-tracing of the cat with a connectivity matrix derived from 1139 connections between 65 cortical areas and again, similarly coded with 1 for weak/sparse up to 3 for strong/dense.

Both analyses of matrices were done using nonmetric multidimensional scaling (NMDS) which plots strongly connected areas in closely while maximising the graphical distance between non-connected areas. The resulting graphs showed strong interconnectivity between the dorsal and ventral areas confirming Felleman and Van Essen in Young et al.'s studies and for Scannell et al., four distinct clusters of strong local interconnectivity were seen - designated visual, auditory, somatosensorimotor and frontolimbic - with relatively sparse connectivity between the clusters themselves. Furthermore, Bassett and Bullmore notes that in a study by Hilgetag et al.[10], *the larger scale macaque and cat cortical connectivity matrices described above were formally shown to have small-world properties, that is, relatively high clustering ( $\gamma \gg 1$ ) and short path lengths ( $\lambda \sim 1$ ) compared to random networks.*

A less biological data driven approach was taken by Postma et al.[15] where they examined the semantic associative network for a set of words and measured the characteristic path length and clustering coefficient at various stages of development. They artificially created a semantic net in two ways by using data taken from the MRC psycholinguistic database containing word-association and age-of-acquisition data published by Michael Wilson<sup>2</sup> online. The connections were defined by a study by Kiss et al.[11] where people were asked to associate freely on target words. The association data were used for two models - first, in combination with age-of-acquisition data where they were translated into age-dependent semantic networks and second, they were trained without the age-of-acquisition data as a hetero-

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<sup>2</sup><http://ling.ed.ac.uk/help/mrc>

associative memory (HAM) model of semantic development.

For the first model the presence of an associative relation between any two concepts is represented by the  $N \times N$  matrix  $\mathbf{A}$ . Each element  $A(t, i, j) = A(t, j, i)$  of the matrix  $\mathbf{A}(\mathbf{t})$  ( $t$  represents age in years  $\times 100$ ) specifies the presence ( $A(t, i, j) = 1$ ) or absence ( $A(t, i, j) = 0$ ) of an association between the concepts  $i$  and  $j$  at time  $t$ . The total number of concepts  $N$  is acquired at age  $t = 700$  and for any concept  $h$  that is not yet acquired, it is represented by  $A(t, h, j) = A(t, j, h) = 0, 1 \leq j \leq N$ . Also, for each target word  $w$ , a response word  $r$  was generated by a proportion  $\lambda(w, r)$  of the subjects. Thus,  $A(700, w, r) = s[\lambda(w, r)]$ , with  $s[x] =$  the step function,<sup>3</sup> and for  $t < 700$ ,  $A(t, w, r) = s[aoa(w) - t] \times s[aoa(r) - t] \times A(800, w, r)$ , with  $aoa(w)$  being the age of acquisition of word  $w$ .

For the second model Postma et al. presented the concepts acquired to a HAM model consisting of  $M$  in and output nodes and  $M^2$  adaptive weights. Associations among concepts were represented by means of clipped Hebbian learning, i.e at each presentation of a pattern-pair, weights are set to  $w(i, j) = \min[1, w(i, j)' + \lambda p(i)q(j)]$ . (Aside: Note that the usage of Hebbian learning is subject to criticism which we shall deal with later.) Nevertheless, Postma et al. sets  $\lambda = 0.2$ , trains the network seven times to simulate the developmental stages and at each stage pairs of concepts are randomly selected with a probability proportional to their association strength. We also note that they do not give any justification for the choice of these numbers, one can only presume that they should be arbitrary for independence of results. They incorporate stochastic synapses to accommodate the association of a single input vector to multiple output vectors which is a modification to traditional HAM modules which associate single input vectors to single output ones. For  $p(j)$  an input vector, elements of output vector  $q(i)$  is defined as:

$$q(i) = \sum_{j=1}^M f(w_{i,j})p(j),$$

with  $f : \mathbb{R} \rightarrow \{0, 1\}, f(x) = 1$ , for  $rand < x, f(x) = 0$  otherwise, and  $rand$  is a random value taken from the uniform distribution on the unit interval.

Thus, the resulting tables are as follows:

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<sup>3</sup>i.e.  $s[x] = 1, x > 0$  and  $s[x] = 0$  otherwise.

$t$	100	200	300	400	500	600	700	adult
$N_1$	-	30	322	728	1066	1185	1205	8210
$N_2$	20	35	266	677	999	1011	1205	-
$k_1$	-	3.2	10.5	14.7	17.0	17.7	17.1	58.7
$k_2$	1.2	3.0	9.4	12.1	16.4	17.0	18.0	-

where the ‘adult’ column specifies  $N_1$  and  $k_1$  for the entire set of concepts for which association data are available for the first model and  $N_2$  and  $k_2$  are associated with the HAM model.

The analysis of the results were then done by calculating the clustering coefficients and path lengths. Postma et al. then compared these values with ones obtained from a random network (denoted by letting  $p = 1$ ) so that they got  $L \approx L(1)$  and  $C \gg C(1)$  which is akin to Watts’ and Strogatz’ criteria from earlier. Overall, the main findings of Postma et al.’s paper are that the characteristic path length remains almost constant despite expansion of the network ( $C \approx 0.13, L \approx 2.7$ ) and that these values indicate small-worldliness. It is again noteworthy that the binary association network produced here are rather crude approximations of the human associative strengths and that the almost stagnant path length didn’t form until the later stages of development ( $\approx \frac{400-500}{100}$  years). However, the conclusion which mentions the apparent tendency of the human semantic network to optimise small-worldliness by maintaining a constant path length and low clustering coefficient despite increasing number of concepts is also discussed in Bassett and Bullmore’s paper from earlier.

Bassett and Bullmore suggest that the small-world brain network topology may have been selected to optimise the problem of cost-efficient information processing. They gave several citations which provide supports for this claim that the anatomical structure of the brain has evolved with a small-world structure for information optimisation such as the segregation of visual cortexes of the cat and macaque monkey as seen earlier. Furthermore, they give recent studies based on electroencephalogram (EEG), magnetoencephalogram (MEG) and functional magnetic resonance imaging (fMRI). To quantify these measures, e.g. using EEG or fMRI, several methods such as recording time series for several regions of the brain and then estimating the functional connectivity between a given pair of regional time series in terms of their mutual information, correlation or partial correlation.[5] Finer details can be found in [5] but for the purpose of this review we don’t need a

deep understanding of these procedures, we need just be aware that the relative merits of these various technical options remains to be fully evaluated, as also noted by Bassett and Bullmore.

According to Bassett and Bullmore, the first graph theoretical analysis of MEG data was reported by Stam[19] who measured the activity from 126 MEG sensors in five people who were in a resting state (eyes closed). Activity between sensors was estimated using synchronisation likelihood after the raw data was filtered into EEG bands of  $\delta, \theta, \alpha, \beta$  and  $\gamma$  whose magnitudes are approximately  $< 4$  Hz, 4-8 Hz, 8-12 Hz, 13-30 Hz and  $>30$  Hz, respectively<sup>4</sup>. The resulting functional connectivity matrices were thresholded to create a set of graphs depicting brain functional network corresponding to each of the frequency bands. Their results showed that the  $\alpha$  and  $\beta$  bands had a regular topology whereas the more extreme  $\delta, \theta$  and  $\gamma$  bands had small-world properties. A similar study was conducted by Micheloyannis et al.[13] in 2006 with 28 sensors and 14 individuals. The analysis was similarly done (using synchronisation likelihood), however, they found evidence for small-world topology on **all** frequency bands. Bassett and Bullmore's conclusion regarding these conflicting results is that *'the currently available data are inconclusive at this point'*.

However, it is noteworthy to us that in the second study Micheloyannis et al. had significantly more participants (14 versus 5) and that the task involved was quite different - a memory test versus Stam et al.'s resting state.

In relation to fMRI, despite the narrow bandwidths suggesting initial limitations for study, appropriate functional connectivity analysis such as wavelet-based introduced by Achard et al.[2] who reported small-world properties of fMRI networks at all frequencies in the range 0.007-0.45 Hz.

In addition, as mentioned in the introduction, the degree distribution of a network can provide information of the processes determining its formation and growth. Bassett and Bullmore also report an fMRI study by Eguíluz et al.[7] who found a power law degree distribution for a functional network of activated voxels. However, the degree distribution of whole-brain fMRI networks of cortical regions has also been described as an exponentially truncated power law[2][5]. This means that the probability of highly con-

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<sup>4</sup>See <http://www.crossroadsinstitute.org/eeg.html> for further explanation of these bands in physical terms.

nected hubs existing in the brain is somewhat less than in a small-world network but is still more likely than a random one. Moreover, Bassett and Bullmore remind us that many complex systems that are physically embedded or constrained also exhibit this truncated power law. Thus the existence of such a distribution in the brain could reflect the constraint of aging of brain regions[6]. Several other examples were also given where the node can ‘age’ i.e. not form new connections, or becomes isolated, i.e. old connections removed. These aspects coincide with Barabási’s incorporation of growth and preferential attachment which gives an indication of a possible fruitful area of research.

## 2.2 Criticisms

This section takes several arguments from Asim Roy’s ‘Artificial neural networks: a science in trouble’ paper[16]. Though the nature of this review is not specifically dealing with neural networks, the form of a small-world network is of a connectionist kind and in particular with regard to our second paper where Postma et al. modelled word association with a HAM model which is a form of artificial neural network. The first major misconception mentioned by Roy is the (mis)assumption that ‘no synaptic change signals are allowed to the cells from other sources within the brain.’ This can be specifically applied critically to Postma et al.’s use of the HAM model which is based on Hebbian learning. Hebbian learning simply assumes that ‘neurons that fire together, wire together’, thereby not having any other further influences.

Likewise, Roy’s explanation of the misconception that the brain learns instantly from each and every learning example presented to it is another flaw in Postma et al.’s HAM model. From everyday experience, it is not difficult to see that human learning is not always instantaneous, but may occur later based on information it has already collected and stored in the brain. In Postma et al.’s HAM model the network was ‘trained’ 7 times to produce the results observed as ‘learning’. According to Roy, this is insufficient to be taken as ‘learning’ as it is ‘memoryless’ (as described in his second misconception). Inputs, weights and functions are updated simultaneously and no information (analogous to memory) is retained which is counterintuitive to the way humans (or their brains) learn. Thus one must take into account these factors which may hinder the plausibility of the small-world

network observation constructed from Postma et al.'s HAM model.

Moreover, though Roy's arguments do not apply directly to Bassett and Bullmore's paper, one must be aware of the incompleteness of the small-world phenomenon results of the visual cortex of the cat and macaque monkey and the various EEG and fMRI studies. In particular, because there are different ways of forming and analysing a connectivity matrix, more specifically the way one encodes the relative strengths of connections can be done less discretely (e.g. instead of 0,1,2, one can have a larger, more continuous range). One cannot disregard the possibility that a different method of analysis may reveal a differently network topology, however unlikely.

### 3 Summary/Conclusion

Overall, as seen throughout this review, there are many reports, in particular from biological data, that indicate the existence of small-worldliness in the structure of the brain. Conversely, we have Postma et al.'s paper where the modelling of human memory association was made and a small-world structure also observed. However, as highlighted in our 'Criticisms' section, caution must be taken due to the use of artificial neural networks to simulate Postma et al.'s results which led to the small-world observation.

Thus, in conclusion, it *does* appear that there is much potential for the brain to be modelled as a small-world network, however, more so 'bottom-up' rather than 'top-down'. That is, it would be more beneficial to research with more concrete, biological data to confirm the small-world phenomena. We say so because, thus far, we have seen that it appears to hold true for the visual cortexes of the cat and the macaque monkey but little, other than somewhat conflicting results with EEG and fMRI scans have been done with regards to the human brain. Furthermore, another avenue of interest may be to invest research into the mathematical foundations as done by Barabási, in particular the possibility of incorporating the notion of 'dying' nodes, which Barabási surprisingly hasn't mentioned. (He only mentions growth, but e.g. in the WWW, it is common for webpages to become inaccessible.)

## 4 Appendix

### 4.1 Mathematical concepts (as defined by Bassett and Bullmore)

From graph theory,  $n$  = number of nodes,  $k$  = number of edges, degree of node = number of edges on each node.

The **degree distance** of a graph is the probability distribution of  $k$  (= degree).

**Random graphs** have exponential degree distribution:  $P(k) \sim \exp^{-\alpha k}$ .

Some **complex systems** have power law distribution:  $P(k) \sim k^{-\alpha}$ , e.g. world-wide web.

Physically embedded networks have exponentially truncated power law distribution of the form:  $P(k) \sim k^{\alpha-1} \exp^{-\frac{k}{k_c}}$ .

**Path Length** between two nodes = the minimal number of edges that must be traversed to form a direct connection between the two nodes of interest.

**Clustering coefficient** of a node is the measure of the number of edges that exist between its nearest neighbours.

**C has maximum normalised value** if all the nearest neighbours of an index node are also nearest neighbours of each other.

**Minimum path length and minimum clustering coefficient of a graph** is estimated by averaging  $L_i$  and  $C_i$  for each of the  $i = 1, \dots, n$  nodes.

In a *random graph*, the average minimum path length is typically *short* and clustering coefficient, *small*:

$$L_{rand} \sim \frac{\ln N}{\ln\left(\frac{k}{N} - 1\right)}$$

$$C_{rand} \sim \frac{K}{N}.$$

For a small-world network:  $\lambda = \frac{L}{L_{rand}} \sim 1$  and  $\gamma = \frac{C}{C_{rand}} > 1$ .

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